

NAG Toolbox for MATLAB

f07us

1 Purpose

f07us solves a complex triangular system of linear equations with multiple right-hand sides, $AX = B$, $A^T X = B$ or $A^H X = B$, using packed storage.

2 Syntax

```
[b, info] = f07us(uplo, trans, diag, ap, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07us solves a complex triangular system of linear equations $AX = B$, $A^T X = B$ or $A^H X = B$, using packed storage.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J 1989 The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates whether A is upper or lower triangular.

uplo = 'U'

A is upper triangular.

uplo = 'L'

A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **trans** – string

Indicates the form of the equations

trans = 'N'

The equations are of the form $AX = B$.

trans = 'T'

The equations are of the form $A^T X = B$.

trans = 'C'

The equations are of the form $A^H X = B$.

Constraint: **trans** = 'N', 'T' or 'C'.

3: **diag** – string

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'

A is a nonunit triangular matrix.

diag = 'U'

A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **ap**(*) – complex array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The n by n triangular matrix A , packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in **ap**($i + j(j - 1)/2$) for $i \leq j$;
if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in **ap**($i + (2n - j)(j - 1)/2$) for $i \geq j$.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced; the same storage scheme is used whether **diag** = 'N' or 'U'.

5: **b**(ldb,*) – complex array

The first dimension of the array **b** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, \text{nrhs_p})$

The n by r right-hand side matrix B .

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The first dimension of the array **ap** and the second dimension of the array **ap**. (An error is raised if these dimensions are not equal.)

n , the order of the matrix A .

Constraint: $n \geq 0$.

2: **nrhs_p** – int32 scalar

Default: The second dimension of the array **b**.

r , the number of right-hand sides.

Constraint: **nrhs_p** ≥ 0 .

5.3 Input Parameters Omitted from the MATLAB Interface

ldb

5.4 Output Parameters

1: **b**(ldb,*) – complex array

The first dimension of the array **b** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, \text{nrhs_p})$

The n by r solution matrix X .

2: **info** – **int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **trans**, 3: **diag**, 4: **n**, 5: **nrhs_p**, 6: **ap**, 7: **b**, 8: **ldb**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = i , $a(i, i)$ is exactly zero; A is singular and the solution has not been computed.

7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham 1989.

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon|A|,$$

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_{\infty}}{\|x\|_{\infty}} \leq c(n) \operatorname{cond}(A, x)\epsilon, \quad \text{provided} \quad c(n) \operatorname{cond}(A, x)\epsilon < 1,$$

where $\operatorname{cond}(A, x) = \| |A^{-1}| |A| |x| \|_{\infty} / \|x\|_{\infty}$.

Note that $\operatorname{cond}(A, x) \leq \operatorname{cond}(A) = \| |A^{-1}| |A| \|_{\infty} \leq \kappa_{\infty}(A)$; $\operatorname{cond}(A, x)$ can be much smaller than $\operatorname{cond}(A)$ and it is also possible for $\operatorname{cond}(A^H)$, which is the same as $\operatorname{cond}(A^T)$, to be much larger (or smaller) than $\operatorname{cond}(A)$.

Forward and backward error bounds can be computed by calling f07uv, and an estimate for $\kappa_{\infty}(A)$ can be obtained by calling f07uu with **norm_p** = 'I'.

8 Further Comments

The total number of real floating-point operations is approximately $4n^2r$.

The real analogue of this function is f07ue.

9 Example

```
uplo = 'L';
trans = 'N';
diag = 'N';
ap = [complex(4.78, +4.56);
      complex(2, -0.3);
      complex(2.89, -1.34);
```

```
        complex(-1.89, +1.15);  
        complex(-4.11, +1.25);  
        complex(2.36, -4.25);  
        complex(0.04, -3.69);  
        complex(4.15, +0.8);  
        complex(-0.02, +0.46);  
        complex(0.33, -0.26)];  
b = [complex(-14.78, -32.36), complex(-18.02, +28.46);  
     complex(2.98, -2.14), complex(14.22, +15.42);  
     complex(-20.96, +17.06), complex(5.62, +35.89);  
     complex(9.539999999999999, +9.91), complex(-16.46, -1.73)];  
[bOut, info] = f07us(uplo, trans, diag, ap, b)
```

```
bOut =  
  -5.0000 - 2.0000i    1.0000 + 5.0000i  
  -3.0000 - 1.0000i   -2.0000 - 2.0000i  
   2.0000 + 1.0000i    3.0000 + 4.0000i  
   4.0000 + 3.0000i    4.0000 - 3.0000i  
info =  
      0
```